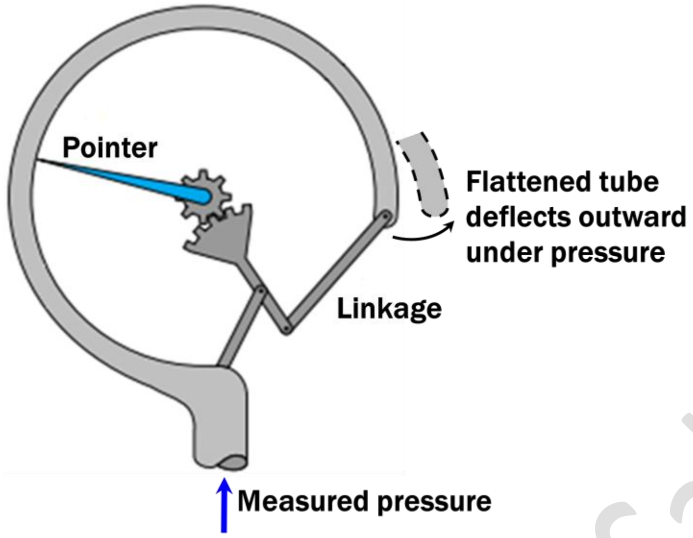
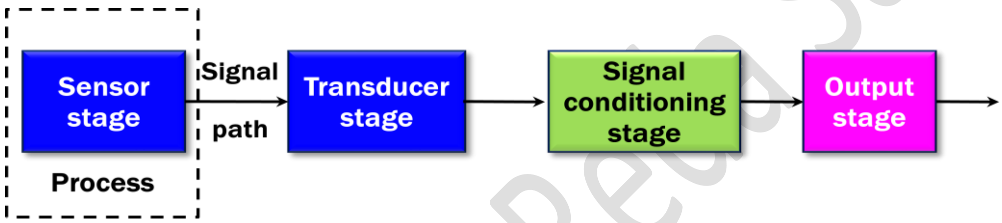
1-a) Identify the main components in the measuring systems of:

(i) C-shaped Bourdon pressure gauge



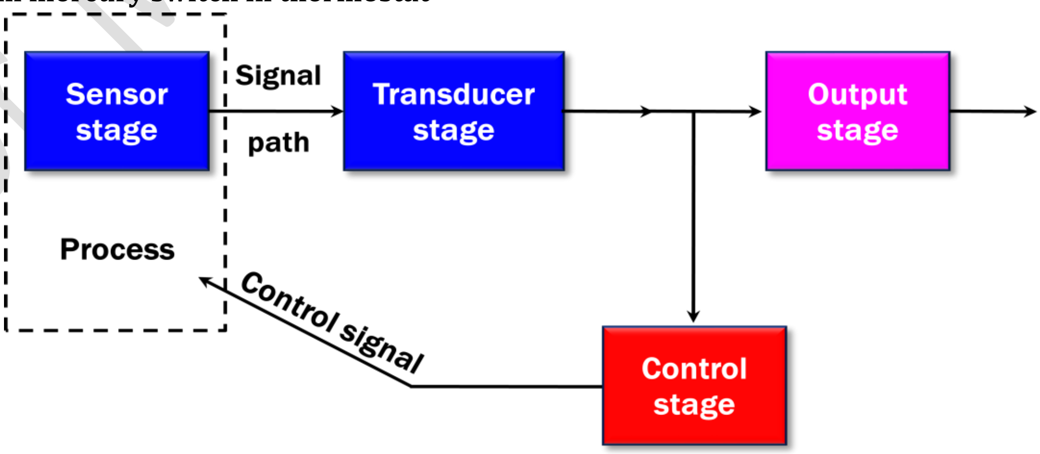


Sensor-transducer stage — **The curved tube** acts as the sensor and transducer, where it senses the measured pressure and transforms it into a detectable mechanical displacement.

Signal conditioning stage — **The gears** condition the signal by amplifying the signal of the curved tube deflection.

Output stage — The **readout scale** serves as the output stage of that measurement system.

(ii) Room mercury switch in thermostat



Sensor-transducer stage — **Bimetallic thermometer** acts as the sensor and transducer, where it senses the measured thermal energy and transforms it into a detectable mechanical displacement.

Output stage — **Displacement of thermometer tip**, as it moves the pointer.

Feed back control stage — **Mercury contact switch** interprets the measured temperature and makes a decision regarding the control of the process.

1-b) Solution:

At atmospheric pressure, the boiling temperature of water, $X_t = 100°C$

Also,

$\varepsilon_i = X_i - X_T = X_i - 100$ $\qquad \therefore X_i = 100 + \varepsilon_i$

$\varepsilon_{i,R}(\%) = \frac{\varepsilon_i}{X_t} * 100$ $\qquad \therefore \varepsilon_i = \frac{\varepsilon_{i,R}(\%) * X_t}{100} = \frac{\varepsilon_{i,R}(\%) * 100}{100} = \varepsilon_{i,R}(\%)$

N_i	1	2	3	4	5	6	7	8	9	10
ε_i	0.8	1.0	0.4	0.2	0.5	−0.1	0.9	0.0	0.4	0.6
X_i	100.8	101	100.4	100.2	100.5	99.9	100.9	100	100.4	100.6

Also,

Deviation $= d_i = X_i - \overline{X}$ $\qquad$ Mean reading $= \overline{X} = \frac{\sum X_i}{N} = \frac{1004.7}{10} = \boxed{100.47°C}$

Therefore,

N_i	1	2	3	4	5	6	7	8	9	10
d_i	0.33	0.53	−0.07	−0.27	0.03	−0.57	0.43	−0.47	−0.07	0.13

Average Deviation $= D = \frac{\sum |d_i|}{N} = \frac{2.9}{10} \cong \boxed{0.29°C}$

Standard Deviation $= \delta = \sqrt{\frac{\sum d_i^2}{N-1}} = \sqrt{\frac{1.221}{9}} \cong \boxed{0.368°C}$

$\therefore$ Variance $= \delta^2 = 0.368^2 \cong \boxed{0.135°C}$

Uncertainty $= \omega_T = \pm\sqrt{\sum d_i^2} = \pm\sqrt{1.221} \cong \boxed{\pm 1.105°C}$

2-a) Define the error of the measurement and its main types.

Measurement Error or absolute error (ε) is the difference between the measured value and true (known standard) value (<u>does not</u> written on the instrument).

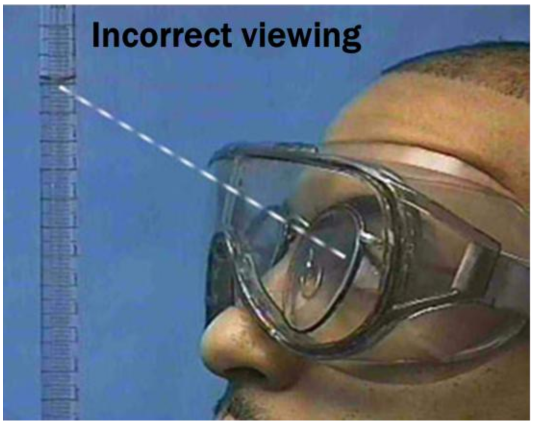
$$\varepsilon = X_{measured} - X_{true}$$

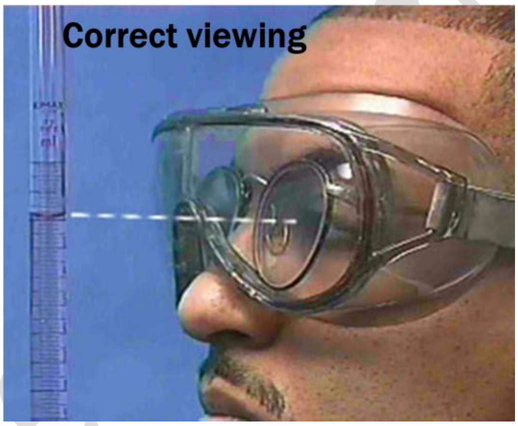
Types of Measurement Errors:

1) Gross Error

Gross errors are basically human errors caused by the person using the instrument. Some reasons for gross errors are:

➔ **Reading with parallax error.**





➔ Improper applications of instruments: Using a 0–100 V voltmeter to measure 0.1 V, etc.

➔ Wrong computation.

2) Systematic Error

Systematic error is a <u>constant</u> deviation of operation in instruments. It causes the measured result to deviate by a <u>fixed</u> amount in <u>one direction</u> from the correct value, and thus may <u>not</u> be reduced by averaging over a lot of data.

A systematic error influences the <u>accuracy</u> of the result.
It can be estimated by comparing your results to other results of another equipment.

Some reasons systematic errors are:

☻ Friction in various moving components.

☻ Irregular spring tension in analog meters.

☻ Calibration errors due to aging.

3) Random Error

Random error is a measure of the random variation found during repeated measurements of a variable.

Therefore, experiments with very small random errors are said to have a high degree of <u>precision</u> (A random error influences the <u>precision</u> of a result).

These errors can only be estimated by statistical analysis.

2-b) Given

$P = \rho R T$

$R = 287\ J/kg.K \pm 0.2\%$

$T = 25 \pm 0.2°C = 298 \pm 0.2\ K$

$P = 105\ kPa = 105000\ Pa$

$\rho = ??\qquad \omega_\rho = ??$

Solution

$$\omega_P = \pm\sqrt{\varepsilon_L^2 + \varepsilon_H^2 + \varepsilon_K^2 + \varepsilon_Z^2} = \pm\sqrt{\left(\frac{0.1 * 100}{100}\right)^2 + \left(\frac{0.1 * 100}{100}\right)^2 + \left(\frac{0.15 * 100}{100}\right)^2 + \left(\frac{0.2 * 100}{100}\right)^2}$$

$$\therefore \omega_P \cong \pm 0.28723\ kPa \cong \pm 287.23\ Pa$$

$$\rho = \frac{P}{RT}$$

$$\therefore \omega_\rho = \pm\sqrt{\left(\frac{\partial \rho}{\partial P}\omega_P\right)^2 + \left(\frac{\partial \rho}{\partial R}\omega_R\right)^2 + \left(\frac{\partial \rho}{\partial T}\omega_T\right)^2}$$

$\omega_P = \pm 287.23\ Pa \qquad \omega_T = \pm 0.2°C \qquad \omega_R = \pm\frac{0.2 * 287}{100} \cong \pm 0.574\ J/kg.K$

$$\frac{\partial \rho}{\partial P} = \frac{1}{RT} = \frac{1}{287 * 298} \cong 1.16924 * 10^{-5} \qquad \frac{\partial \rho}{\partial R} = \frac{-P}{R^2 T} = \frac{-100000}{287^2 * 298} \cong -0.004074$$

$$\frac{\partial \rho}{\partial T} = \frac{-P}{RT^2} = \frac{-100000}{287 * 298^2} \cong -0.003924$$

$$\therefore \omega_\rho = \pm\sqrt{(1.16924 * 10^{-5} * 287.23)^2 + (0.004074 * 0.574)^2 + (0.003924 * 0.2)^2} \cong \boxed{\pm 0.00417\ kg/m^3}$$

$$\rho = \frac{P}{RT} = \frac{100000}{287 * 298} \cong \boxed{1.169\ kg/m^3} \qquad \therefore \boxed{\rho \cong 1.169 \pm 0.00417\ kg/m^3 \cong 1.169\ kg/m^3 \pm 0.3564\%}$$

<u>Another Solution</u>

$$\rho = \frac{P}{RT} = \frac{100000}{287 * 298} \cong \boxed{1.169\ kg/m^3} \qquad \rho = \frac{P}{RT} = P\, R^{-1}\, T^{-1}$$

$$\frac{\omega_\rho}{\rho} = \pm\sqrt{\left(\frac{1 * \omega_P}{P}\right)^2 + \left(\frac{-1 * \omega_R}{R}\right)^2 + \left(\frac{-1 * \omega_T}{T}\right)^2}$$

$\omega_P = \pm 287.23\ Pa \qquad \omega_T = \pm 0.2°C \qquad \frac{\omega_R}{R} = \pm 0.2\%$

$$\therefore \frac{\omega_\rho}{\rho} = \pm\sqrt{\left(\frac{287.23}{100000}\right)^2 + (0.002)^2 + \left(\frac{0.2}{298}\right)^2} \cong \pm 0.003564$$

$$\therefore \omega_\rho = \pm 0.003564 * 1.169 \cong \boxed{\pm 0.00417\ kg/m^3 \cong \pm 0.3564\%}$$

$$\therefore \boxed{\rho \cong 1.169 \pm 0.00417\ kg/m^3 \cong 1.169\ kg/m^3 \pm 0.3564\%}$$

3-a) Synthesis the following terms as applied to measurement system:

Variable: Variables are entities that influence the test and affect the outcome as T, P, u...

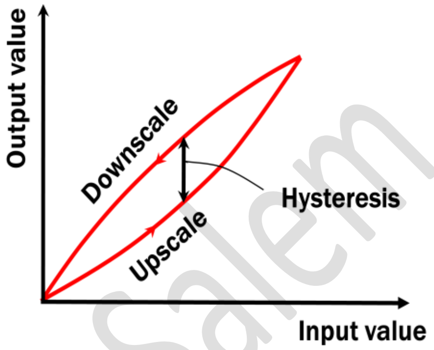
- Independent Variable; it changes independently of other variables.
- Dependent Variable; it is affected by changes in one or more other variables.

Parameter: Parameter is a group of variables as Re, Gr, Pr, ... (example: Re = ud/υ)

Hysteresis error: It refers to the maximum difference for the same measured quantity between the upscale sequential test and a downscale sequential test.

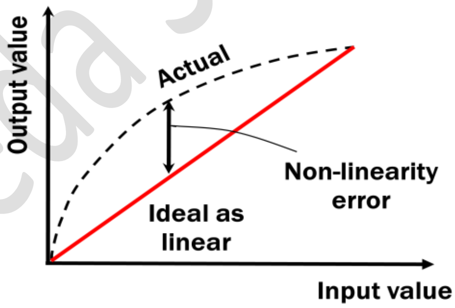
It may be due to mechanical friction, magnetic effects, elastic deformation or thermal effects.

$$\varepsilon_H = y_{downscale} - y_{upscale}$$



Non-linearity error: Non-Linearity error is the maximum difference between the actual data and the ideal linear relation between input and output.

$$\varepsilon_L(x) = y(x) - y_L(x)$$

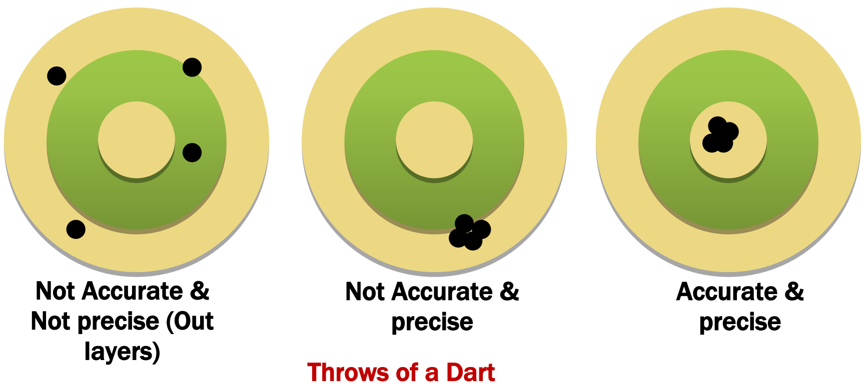


Sensitivity: Sensitivity indicates how much the output of an instrument system changes when the quantity being measured changes by a given amount.

$$\text{sensitivity} = K = \frac{\text{change of output reading}}{\text{change of input}} = \frac{\Delta X_o}{\Delta X_i}$$

Accuracy: Accuracy is the closeness of a measured value to the true value being measured. In other words, it is the minimum graduation (reading) that can be taken from the measuring instrument.

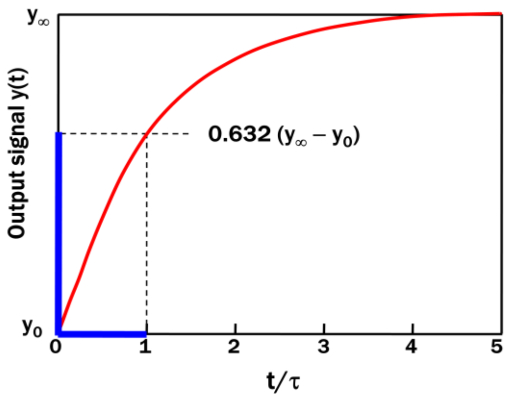
Precision: Precision is the ability of a measuring instrument to reproduce a certain reading with a given accuracy. These readings may or may not be accurate, but will repeat. The term precision is used to describe the degree of freedom of a measurement system from random errors.

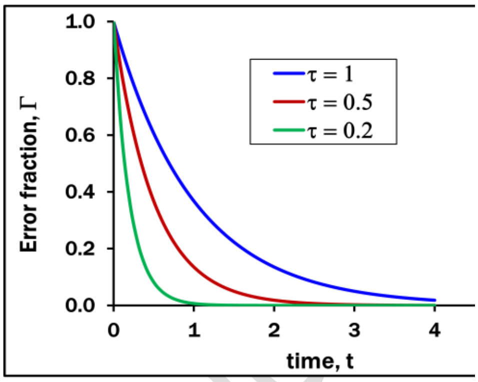


Throws of a Dart

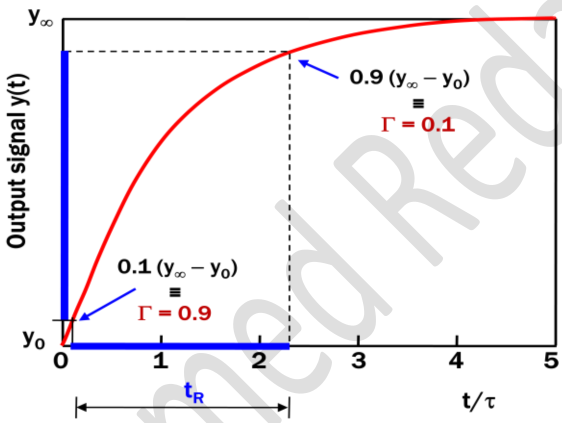
Uncertainty: It characterizes the range of values within which the true value is asserted to lie. It is written on the instrument

Time constant: It is the time required for a system to achieve 63.2% of the step change magnitude ($y_\infty - y_0$). It is a measure of the speed of system response.





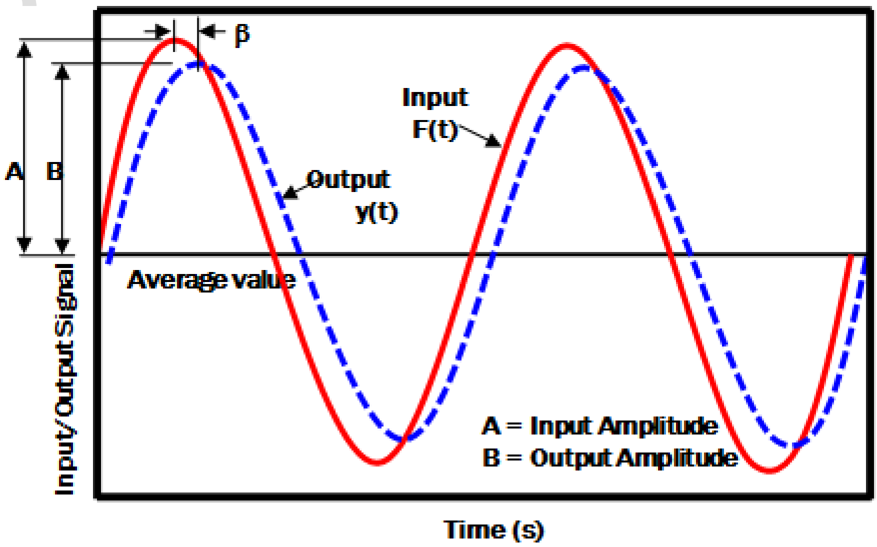
Rise time: Rise time of a system is the length of time required for the step response to rise from 0.1(10%) to 0.9(90%) of the step change magnitude ($y_\infty - y_0$).



$$t_R = t_{\Gamma=0.1} - t_{\Gamma=0.9}$$

Time lag: Time lag is the time interval between the maximum force input and maximum displacement output.

$$\beta = \frac{\emptyset}{\omega}$$

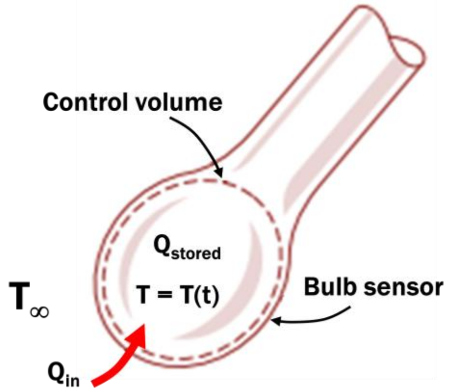


3-b) Given

First – order system $\quad T_0 = 25°C$

$T_\infty = 65°C \quad T(t) = ??$

$\tau = 25\ ms = 0.025\ s \quad t_R = ??$



Bulb of a thermometer

Solution

Apply energy balance for a body: $\therefore Q_{in} - Q_{out} + Q_g = Q_{st}$

For the bulb of the thermometer, we can assume that no heat out and no heat generation

$\therefore Q_{out} = Q_g = 0$

$\therefore Q_{in} = Q_{st} \qquad \therefore \bar{h}A_s[T_\infty - T(t)] = \rho VC\dfrac{dT}{dt}$

$\therefore T_\infty - T(t) = \dfrac{\rho VC}{\bar{h}A_s}\dfrac{dT}{dt} \qquad \therefore T_\infty - T(t) = \tau\dfrac{dT}{dt} \quad$ where $\quad \tau = \dfrac{\rho VC}{\bar{h}A_s}$

Let $\theta = T_\infty - T(t) \qquad \therefore \dfrac{d\theta}{dt} = \dfrac{-dT}{dt}$

$\therefore \theta = -\tau\dfrac{d\theta}{dt} \qquad \therefore \dfrac{d\theta}{\theta} = \dfrac{-1}{\tau}dt$

$\therefore \displaystyle\int_{\theta_0}^{\theta(t)}\frac{d\theta}{\theta} = \frac{-1}{\tau}\int_0^t dt \qquad \therefore \ln\theta(t) - \ln\theta_0 = \ln\frac{\theta(t)}{\theta_0} = \ln\frac{T_\infty - T(t)}{T_\infty - T_0} = \frac{-t}{\tau}$

$\therefore \dfrac{T(t) - T_\infty}{T_0 - T_\infty} = e^{-t/\tau} \qquad \therefore T(t) = T_\infty + (T_0 - T_\infty)e^{-t/\tau} = 65 + (25 - 65)e^{-t/0.025}$

$\therefore \boxed{T(t) = 65 - 40e^{-40\,t}}$

$\Gamma = 0.1 = e^{-40\,t_{\Gamma=0.1}} \qquad \therefore t_{\Gamma=0.1} \cong 0.05756\ s = 57.56\ ms$

$\Gamma = 0.9 = e^{-40\,t_{\Gamma=0.9}} \qquad \therefore t_{\Gamma=0.9} \cong 0.00263\ s = 2.63\ ms$

$\therefore t_R = t_{\Gamma=0.1} - t_{\Gamma=0.9} = 57.56 - 2.63 \cong \boxed{54.93\ ms}$